

Deconstructing Dimensional Deconstruction

Kenneth Lane
Department of Physics
Boston University
590 Commonwealth Avenue
Boston, MA 02215, USA*

Dimensional deconstruction (DD) abstracts from higher dimensional models features of related 4-dimensional ones. DD was proposed in Refs. [1–3] as a scheme for constructing models of *naturally* light composite Higgs boson. These are models in which—*without fine-tuning of parameters*—the composite Higgs’s mass M and vacuum expectation value v are much lighter than its binding energy scale Λ . We review the basic idea of DD. It is easy to arrange $M \ll \Lambda$. We show, however, that DD fails to give $v \ll \Lambda$ in a model that is supposed to contain a naturally light composite Higgs [4].

1. WHAT IS DIMENSIONAL DECONSTRUCTION?

There has been considerable interest lately in a new approach to model-building called “dimensional deconstruction” (DD). In the beginning, there were two views of DD. The one we discuss in this paper is due to Arkani-Hamed, Cohen and Georgi (ACG) [1,2]. It is based on the fact that certain renormalizable, asymptotically free 4d field theories look, *for a limited range of energies*, like $d > 4$ -dimensional theories in which the extra dimensions are compactified and discretized (on a periodic lattice). Here, the extra dimensions are a mirage. The other view is that of Hill and his collaborators [3] who assume the extra dimensions are real. They discretize the extra dimensions too—to regulate the theory. Both Arkani-Hamed *et al.* and Hill *et al.* use features of the higher dimensional model to deduce the form, magnitude, and sensitivity to high-scale (Λ) physics of phenomenologically important operators such as mass terms (generically, M), self-interactions (λ), and vevs (v) of light *composite* Higgs bosons (LCH) [5]. Their LCH models aim for $M \simeq v \simeq 100\text{--}200$ GeV and $\Lambda \simeq 10$ TeV, relevant to electroweak symmetry breaking.

The simplest DD example is the $d = 5$ “moose ring” model [1] depicted in Fig. 1. This shows the full content (“UV-completed”) of the model at the high energy scale Λ . It contains N strong $SU(n)$ and weak $SU(m)$ (coupling $g^2/4\pi \ll 1$) gauge groups, with matter fields that are the massless chiral fermions

$$\psi_{Lk} \in (n, m, 1), \quad \psi_{Rk} \in (n, 1, m) \\ \text{of } (SU(n)_k, SU(m)_k, SU(m)_{k+1}). \quad (1)$$

The index k is periodically identified with $k + N$. As $g \rightarrow 0$, these fermions have a large chiral symmetry, $[SU(m)_L \otimes SU(m)_R]^N$. At Λ , the strong $SU(n)$ interactions cause them to condense, creating N sets of $m^2 - 1$ *composite* Goldstone bosons (GBs), π_k^a with $k = 1, \dots, N$ and $a = 1, \dots, m^2 - 1$. Their decay constant $f \simeq \Lambda/4\pi$.

Below Λ , this is a nonlinear sigma model, with fields $U_k = \exp(i\pi_k^a t_a/f) \equiv \exp(i\pi_k/f)$ interacting with the weakly-coupled $SU(m)_k$ gauge fields $A_{k\mu} = A_{k\mu}^a t_a$. They transform as $U_k \rightarrow W_k U_k W_{k+1}^\dagger$ with $W_k \in SU(m)_k$. This low energy theory is represented by the “condensed moose” obtained from Fig. 1 by erasing the $SU(n)$ squares and linking the $SU(m)_k$ and $SU(m)_{k+1}$ circles by U_k .

Now, $N - 1$ gauge boson multiplets eat $N -$

*Email: lane@bu.edu

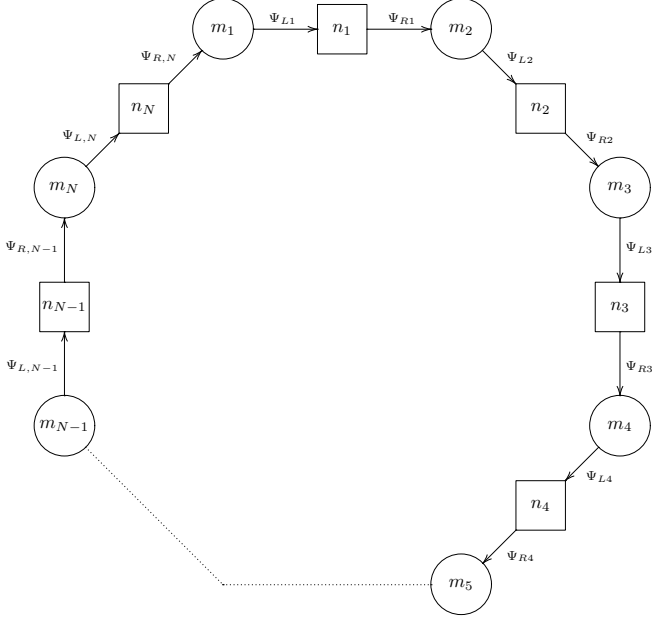


Figure 1. The full moose for the ring model of Ref. [1], showing its UV completion. Strong gauge groups are labeled by n_1, \dots, n_N and weak gauge groups by m_1, \dots, m_N .

1 sets of GBs and acquire the masses $\mathcal{M}_k = 2gf \sin(k\pi/N)$ for $k = 1, \dots, N$. The massless gauge field $A_\mu^a = (A_{1\mu}^a + \dots + A_{N\mu}^a)/\sqrt{N}$ couples with strength g/\sqrt{N} and the uneaten GB is $\pi^a = (\pi_1^a + \dots + \pi_N^a)/\sqrt{N}$. In the unitary gauge, the 4d theory below Λ is described by uniform link variables $U_k = \exp(i\pi^a t_a/\sqrt{N}f)$ plus the massless and massive gauge fields.

Alternatively, at energies well below gf , this looks like a 5d gauge theory: The fifth dimension is compactified on a discretized circle, represented exactly by the *condensed* moose. For $k \ll N$, there is a Kaluza–Klein tower of gauge excitations with masses $\mathcal{M}_k = 2\pi gfk/N$ [1]. The circumference of the circle is $R = Na$ where the lattice spacing $a = 1/gf$ and the 5-dimensional gauge coupling is $g_5^2 = g^2a$. The fifth component of the gauge boson $A_5^a = g\pi^a/\sqrt{N}$. The geometrical connection is clear: π^a is the zero mode associated with rotation about the circle of $SU(m)$ groups in four dimensions and it cor-

responds to the fifth-dimensional gauge freedom associated with A_5^a . At higher energies, $\sim f$ or Λ , the fifth dimension is deconstructed as the underlying asymptotically free 4d theory appears.

2. WHAT IS DD GOOD FOR?

But, π^a is really a 4d pseudoGoldstone boson (PGB) whose symmetry is explicitly broken by the weak $SU(m)_k$ interactions. So it might be a candidate for the LCH of electroweak symmetry breaking. To be a truly *natural* LCH, its vev $v \ll \Lambda$ also. This requires its quartic couplings $\lambda \simeq M^2/v^2 = \mathcal{O}(1)$ or, at least, not $\ll 1$. The idea of DD is that the magnitude and Λ -dependence of M^2 and λ can be deduced from the higher dimensional theory. Let's see.

Higher dimensional gauge invariance allows mass for A_5 from $|\mathcal{W}|^2$, where $\mathcal{W} = P \exp(i \int dx_5 A_5)$ is the nontrivial Wilson loop around the fifth dimension [2]. Since $|\mathcal{W}|^2$ is a nonlocal operator, it cannot be generated with a UV-divergent coefficient. On the discretized circle, $\mathcal{W} = \text{Tr}[\prod_{k=1}^N \exp(iaA_{5k})]$. In the 4d theory this is just the gauge-invariant $\text{Tr}(U_1 U_2 \dots U_N)$, and so this is what provides the mass for π^a . Standard power counting indicates that the strength of $|\text{Tr}(U_1 U_2 \dots U_N)|^2$ is $\Lambda^2 f^2 (g^2/16\pi^2)^N$. This is correct only for $N = 1$. For $N \geq 2$ infrared singularities from the gauge boson masses at $g \rightarrow 0$ overcome this power counting. For $N = 2$, $M^2 \sim g^4 f^2 \log(\Lambda^2/\mathcal{M}_B^2) \sim g^4 f^2 \log(N^2/g^2)$ where $\mathcal{M}_B^2 \sim g^2 f^2/N^2$ is a typical $SU(m)$ gauge boson mass. For $N \geq 3$, $M^2 \sim g^4 f^2$. Thus, for $N \geq 2$ and $g^2/4\pi \sim 10^{-2}$, we have $M \ll \Lambda$, as desired.

DD predicts that π^a will fail as an LCH because the quartic interactions of A_5 are derivatively coupled and/or induced by weak $SU(m)$ interactions. This is true for π^a as well. Since $p/f \sim M/f \sim g^2$, all quartic couplings of π^a are $\leq \mathcal{O}(g^4)$. So, in this model, DD is a reliable guide. To achieve larger λ , ACG applied DD to a 6d model with nonderivative PGB interactions [2].

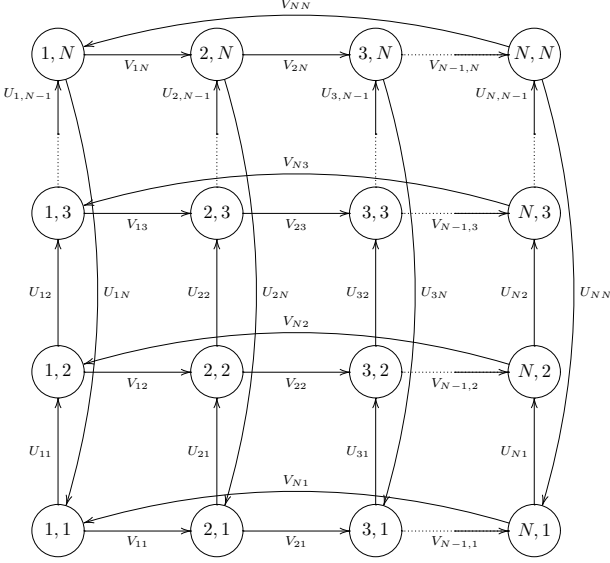


Figure 2. The condensed moose for the 6d toroidal model of Ref. [2].

3. THE 6d TOROIDAL MOOSE MODEL.

Consider a 4d theory described below its UV-completion scale Λ by the condensed moose in Fig. 2. This resembles a discretized torus with $N \times N$ sites labeled periodically by integers (k, l) . Weakly-coupled (g) gauge groups $SU(m)_{kl}$ at the sites are linked by nonlinear sigma model fields U_{kl} and V_{kl} , transforming as

$$\begin{aligned} U_{kl} &= \exp(i\pi_{u,kl}/f) \rightarrow W_{kl} U_{kl} W_{k,l+1}^\dagger, \\ V_{kl} &= \exp(i\pi_{v,kl}/f) \rightarrow W_{kl} V_{kl} W_{k+1,l}^\dagger. \end{aligned} \quad (2)$$

The $\pi_{u,kl}$ and $\pi_{v,kl}$ comprise $2N^2$ $SU(m)$ adjoints of composite Goldstone bosons.

The $SU(m)_{kl}$ gauge bosons eat $N^2 - 1$ sets of GBs. The spectrum of massive gauge bosons, $\mathcal{M}_{kl}^2 = 4g^2 f^2 [\sin^2(k\pi/N) + \sin^2(l\pi/N)]$ is KK-like for small k, l . The massless gauge boson is $B_{NN}^\mu = N^{-1} \sum_{k,l} A_{kl}^\mu$ and its coupling is g/N . Among the $N^2 + 1$ leftover PBS, two that ACG proposed as light composite Higgses are

$$\pi_u = \frac{1}{N} \sum_{k,l} \pi_{u,kl}, \quad \pi_v = \frac{1}{N} \sum_{k,l} \pi_{v,kl}. \quad (3)$$

These are the zero modes associated with going around the torus in the U and V -directions.

What does DD predict for the masses and couplings of $\pi_{u,v}$? Viewing the condensed moose as the compactified and discretized dimensions 5,6 of a 6d gauge theory, the extra-dimensional gauge fields are $A_{5,6}^a = g\pi_{u,v}^a/N$. As before, DD predicts small $M_{\pi_{u,v}}^2 \sim g^4 f^2 \log(N^2/g^2)$ for $N = 2$ and $g^4 f^2$ for $N \geq 3$.

In the 6d model, $A_{5,6}$ have moderately strong nonderivative interactions [2]. They come from the term $\text{Tr} F_{56}^2 = \text{Tr}([A_5, A_6]^2) + \dots = \lambda \text{Tr}([\pi_u, \pi_v]^2) + \dots$ which, in turn, arises from the “plaquette” Hamiltonian

$$\mathcal{H}_P = \sum_{k,l} \lambda_{kl} f^4 \text{Tr} \left(U_{kl} V_{k,l+1} U_{k+1,l}^\dagger V_{kl}^\dagger \right) + \text{h.c.} \quad (4)$$

Note that \mathcal{H}_P leaves $\pi_{u,v}$ massless.

In 6d, the quartic coupling may be shown to be $\lambda \equiv \frac{1}{2} \sum_{k,l} \lambda_{kl} / N^4 = g^2 / 2N^2$ [4]. Depending on the N -dependence of the Higgs masses, this may be large enough to give a Higgs vev comparable to $M_{\pi_{u,v}}$. In 4d, this prediction of DD fails. The strength of λ depends entirely on the nature of the toroidal moose model’s UV completion.

The most natural UV completion of this model is the analog of Fig. 1: At Λ , there are $2N^2$ massless fermions ψ with strong $SU(n)$ interactions located midway between the weak $SU(m)$ ’s [4]. Then, the plaquette interaction arises only from weak gauge interactions. It is of $\mathcal{O}(g^4)$ and, so, $v^2 \sim M_{\pi_{u,v}}^2 / \lambda \gg M_{\pi_{u,v}}^2$.

It is possible to find UV completions of the toroidal moose that yield larger λ . They involve elementary scalars and, therefore, supersymmetry to avoid unnatural fine-tuning of parameters [4,6]. More generally, one can construct sigma models whose symmetries are tailored to give an effective Lagrangian with *arbitrarily and separately tunable* M^2 and λ —at least at the one-loop level. This is the basis of an interesting new direction that has evolved from DD [7,8]. But this approach, called “little Higgs”, has nothing to do with the original idea of deconstruction—that the strengths of a composite Higgs’ mass and interactions may be deduced from corresponding terms in higher dimensional gauge theories. Finding a truly dynamical, natural way of UV-completing

little Higgs models remains one of the greatest challenges to this new idea for electroweak symmetry breaking.

Acknowledgements

This is the written version of my talk at The 31st International Conference on High Energy Physics, Amsterdam, The Netherlands, July 24–31, 2002. I thank the organizers for their kind help and solicitude. I have benefitted greatly from discussions with Nima Arkani-Hamed, Bill Bardeen, Sekhar Chivukula, Andy Cohen, Estia Eichten, Howard Georgi and Chris Hill. I thank Gerard 't Hooft and the theory group at the University of Utrecht for their hospitality and lively discussion. I am grateful to Fermilab and its Theory Group for a 2001–2002 Frontier Fellowship which supported this research. It was also supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91ER40676.

REFERENCES

1. N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001); hep-ph/0104005.
2. N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. **B513**, 232 (2001) and hep-ph/0105239 v4; N. Arkani-Hamed, *et al.*, JHEP 0208:020 (2002); hep-ph/0202089.
3. C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. **D64**, 105005 (2001); hep-th/0104035; H. C. Cheng, C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. **D64**, 065007 (2001); hep-th/0104179; H. C. Cheng, C. T. Hill and J. Wang, Phys. Rev. **D64**, 095003 (2001); hep-ph/0105323.
4. This talk is largely based on my paper, K. Lane, *A Case Study in Dimensional Deconstruction*, Phys. Rev. **D65**, 115001 (2002); hep-ph/0202093.
5. D. B. Kaplan and H. Georgi, Phys. Lett. **B136**, 183 (1984); S. Dimopoulos *et al.*, Phys. Lett. **B136**, 187 (1984).
6. A. Cohen, H. Georgi, communications.
7. N. Arkani-Hamed, *et al.*, JHEP 0208:021 (2002), hep-ph/0206020 and JHEP 0207:034 (2002), hep-ph/0206021.
8. M. Schmaltz, *Beyond the Standard Model (theory)*, invited talk at the 31st International Conference on High Energy Physics, Amsterdam, The Netherlands, July 24-31, 2002.